PILOT LEARNING
CALCULUS II ENGINEERING
PROBLEM-SET 9
FALL 2019

(1) Consider the following initial value problem

$$y' = x^2 - 1 - y; \quad y(1) = 0$$

   (a) Graph the direction field and use it to sketch the graph of the solution to the initial value problem.
   (b) Find the exact solution to the initial value problem and verify the graph in part (a).

(2) Solve the following initial value problems
   (a) $y' = x^2y - xy - x^2 - x; \quad y(0) = 2$
   (b) $y' + y\cos x = (\sin x)(\cos x); \quad y(\frac{\pi}{2}) = 1$

(3) Consider the parametric curve

$$x(t) = t^3 - 3\pi t^2 + 3\pi^2 t; \quad y(t) = 1 - \cos t; \quad 0 \leq t \leq 2\pi$$

   (a) Find the slope of the curve as a function of the parameter t
   (b) Find the points on the curve where the tangent lines are horizontal or vertical.
   (c) Sketch the parametric curve.

(4) Find the length of the parametric curve

$$x(t) = 2t^3 - 1; \quad y(t) = 3t^3; \quad -1 \leq t \leq 1$$

(5) Find the area of the region enclosed by the parametric curve

$$x(t) = t^2; \quad y(t) = \frac{1}{t^4 + 1}$$

and the x-axis.

(6) Sketch and find the area enclosed by the following polar curve

$$r = 2 + \cos(3\theta)$$

(7) Sketch and find the length of the following polar curve

$$r = \theta^2 - 1; \quad 0 \leq \theta \leq \pi$$

(8) Determine whether each of the following sequences converges, and if so, find the limit
   (a) $\lim_{n\to\infty} \frac{1}{n^2}$
   (b) $\lim_{n\to\infty} \frac{\ln(n)}{n}$
   (c) $\lim_{n\to\infty} \frac{n^n}{n!}$
   (d) $\lim_{n\to\infty} \sin(n)\sin(1/n)$

(9) Determine whether each of the following series converges, and if so, find the limit
   (a) $\sum_{n=1}^{\infty} \frac{n^2+3}{n^2-3}$
   (b) $\sum_{n=1}^{\infty} \frac{4^{n+4}}{e^n}$